

## Ratios and Proportions

A **ratio** is a comparison between two numbers (with the same units) by division. The ratio of  $x$  and  $y$  can be expressed in the following ways (in simplest form):

$$\frac{x}{y} \quad x:y \quad x \text{ to } y$$

A **proportion** is an equation stating that two ratios are equal.

### Means-Extremes Property of Proportion

$$a:b = c:d$$

This property allows you to determine whether you have a proportion, or solve a proportion equation.

**Words:** product of means  
= product of extremes

**Symbols:**  ~~$\frac{a}{b} \times \frac{c}{d}$~~  then  $ad = bc$

## Example 1: Determine Whether Ratios Form a Proportion

Determine whether the following ratios form a proportion.

a.)  $\frac{7}{8}$  and  $\frac{49}{56}$

$$7(56) = 392 \quad \checkmark$$
$$8(49) = 392$$

Or  $\frac{49}{56} = \frac{7}{8}$

b.)  $\frac{0.25}{0.6}$  and  $\frac{1.25}{2}$ .

c.)  $\frac{4}{5}$  and  $\frac{16}{20} = \frac{4}{5}$

.5 = .75  
Noooo...  
No

Yes

## Example 2: Solve a Proportion

Solve the following proportions.

a.)  $\frac{n}{12} \neq \frac{3}{8}$

$$8n = 3(12)$$

$$8n = 36$$

$$n = \frac{36}{8} = \boxed{\frac{9}{2}}$$

b.)  $\frac{5}{3} = \frac{6}{x+2}$

$$\cancel{5}(x+2) = 3(\cancel{6})$$

$$5x + 10 = 18$$

$$5x = 8$$

$$\boxed{x = \frac{8}{5}}$$

c.)  $\frac{5}{12} = \frac{10}{x-1}$

$$5x - 5 = 120$$

$$5x = 125$$

$$x = \dots$$

$$\boxed{x = 25}$$

## Rates and Scales

A **rate** is a comparison between two numbers by division. In a rate, the two numbers compared must have different units. The ratio of  $x$  and  $y$  can be expressed in the following ways (in simplest form):

$$\frac{x}{y} \quad x \text{ per } y$$

A **scale** is used when making a model or drawing of something that is too large or too small to be conveniently drawn at actual size. A scale compares the model to the actual size using a proportion.

### Example 3: Use Rates

a.) **BICYCLING.** Trent goes on a 30-mile bike ride every Saturday. He rides the distance in 4 hours. At this rate, how far can he ride in 6 hours?

$$\frac{30 \text{ miles}}{4 \text{ hrs}} = \frac{x \text{ miles}}{6 \text{ hrs}}$$

$$30(6) = 4x$$

$$180 = 4x$$

$$x = 45 \text{ miles}$$

**b.) BICYCLING.** The gear on a bicycle is 8:5. This means that for every 8 turns of the pedals, the wheel turns 5 times. Suppose the bicycle wheel turns about 2435 times during a trip. How many times would you have to crank the pedals during the trip?

$$\frac{8 \text{ pedals}}{5 \text{ wheels}} = \frac{x \text{ pedal}}{2435 \text{ wheels}}$$

$$x = 3896$$

#### Example 4: Use a Scale Drawing

**MAP.** In a road atlas, the scale for the map of Connecticut is 5 inches = 41 miles. The scale for the map of Texas is 5 inches = 144 miles. What are the distances in miles represented by  $2\frac{1}{2}$  inches on each map?

Texas

$$\frac{5 \text{ in}}{144 \text{ mi}} = \frac{2.5 \text{ in}}{\text{?}}$$

CT

$$\frac{5 \text{ in}}{41 \text{ mi}} = \frac{2.5 \text{ in}}{x}$$