a.a1a Notes on Simplifying Radicals Name_____

$\sqrt{\ }$ means the "positive square root" of a number.

Consider $\sqrt{25}$. This means the "positive square root" of 25. To find it, ask yourself, "What positive number times itself equals 25?"

Evaluate.

1. $\sqrt{49}$ 2. $\sqrt{100}$ 3. $\sqrt{1}$ 4. $\sqrt{144}$ 5. $\sqrt{0}$

A **radical** is any quantity with a radical symbol, $\sqrt{\ }$.

A **radical expression** is any expression that contains a radical.

The goal of this entire unit is to learn how to simplify radicals. To **simplify radicals** means to perform every operation possible and to make the radicand(s) as small as possible.

In order to accomplish the second part of the starred statement above, we will rely heavily on the product property of radicals:

The Product Property of Radicals

For example, $\sqrt{15} = \sqrt{3} \bullet \sqrt{5}$. Also, $\sqrt{26} = \sqrt{2} \bullet \sqrt{13}$.

Method #1 for Simplifying the Radicand - Perfect Squares

Once again, one of the goals in simplifying radicals is to make the radicand as small as possible.

Consider $\sqrt{12}$. Using the property above:

However, we know

This simplification was made possible because we knew a perfect square (4), divided evenly into the radicand (12).

If a person had written $\sqrt{12} = \sqrt{6} \bullet \sqrt{2}$, then no simplifying could be done, because 6 and 2 are not perfect squares.

Thus, the critical part is that one must choose factors that are **perfect squares**.

Simplify $\sqrt{45}$. Ask yourself, "Which of the perfect squares above divides evenly into 45?"

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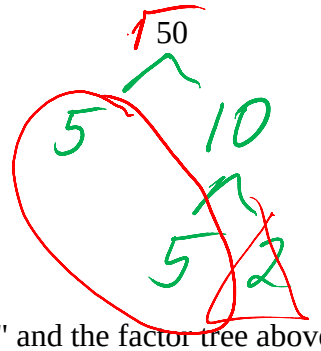
6. Simplify $\sqrt{72}$. $= \sqrt{9} \cdot \sqrt{8}$
$= 3\sqrt{8} = 3 \cdot \sqrt{4} \cdot \sqrt{2}$
$= 3 \cdot 2 \cdot \sqrt{2} = 6\sqrt{2}$

7. Simplify $\sqrt{42}$. $= \sqrt{6} \cdot \sqrt{7}$
$= \sqrt{42}$

Method #2 for Simplifying the Radicand - Twins and a Factor Tree

Create a factor tree for 50:



$5\sqrt{2}$

Apply the story about the "twins" and the factor tree above in order to simplify $\sqrt{50}$.

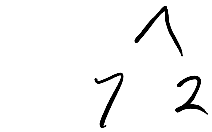
why?: $\sqrt{50} = (50)^{1/2} = (5 \cdot 5 \cdot 2)^{1/2} = (5^2 \cdot 2)^{1/2} 5 \cdot 2^{1/2}$

If you want to use this method, you should always remember:

1)As soon as a number kills its twin, it goes outside of the house **IMMEDIATELY**.
2)If a number has no twins to kill, it must stay inside the house.
3)All of the numbers inside and outside of the house are multiplied together in the end.

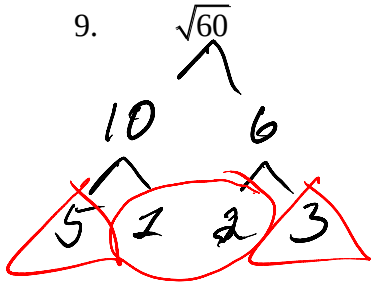
Simplify.

8. $\sqrt{14}$



$\sqrt{7 \cdot 2} = \sqrt{14}$

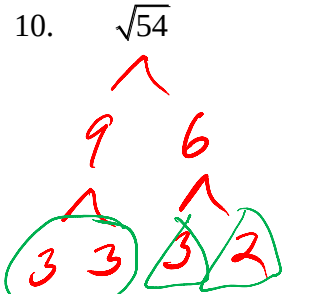
9. $\sqrt{60}$



$2\sqrt{15 \cdot 3}$

$2\sqrt{15}$

10. $\sqrt{54}$



$3\sqrt{6}$

When there are variables in the radicand, it is assumed that they represent positive values. In this situation, the "twins and a factor tree" method is very handy.

Nevertheless, consider $\sqrt{x^{40}}$. What quantity, times itself, equals x^{40}? Remember you add exponents when multiplying.

$\sqrt{x^{40}} = x^{20} \rightarrow (x^{40})^{1/2} \rightarrow x^{20}$

Simplify.

11. $\sqrt{x^2}$

x

12. $\sqrt{y^3z^4}$

$yz^2\sqrt{y}$

13. $\sqrt{3a^{22}}$

$a^{11}\sqrt{3}$

Notes on Multiplying Radicals Name_____

How to Multiply Radicals

- multiply outside numbers
- multiply inside numbers
- simplify radical

For example, consider $3\sqrt{2} \bullet 4\sqrt{5}$.

Multiply "3 and 4", and multiply "2 and 5" to get

$12\sqrt{10}$

Simplify.

14. $9\sqrt{2} \bullet 5\sqrt{6}$

$45\sqrt{12}$

4 3

2 2

$45 \cdot 2\sqrt{3} = 90\sqrt{3}$

15. $(4\sqrt{3})(-8\sqrt{15})(2\sqrt{2})$

$-64\sqrt{90}$

9 10

3 3 5 2

$-192\sqrt{10}$

Homework on Simplifying and Multiplying Radicals

Simplify. Assume all variables represent positive numbers.

1. $\sqrt{64}$
2. $\sqrt{4}$
3. $\sqrt{169}$
4. $\sqrt{256}$

5. $\sqrt{24}$
6. $\sqrt{48}$
7. $\sqrt{32}$
8. $5\sqrt{98}$

9. $-3\sqrt{3}$
10. $10\sqrt{200}$
11. $\sqrt{y^2}$
12. $\sqrt{a^2b}$

13. $\sqrt{c^4d^5}$
14. $\sqrt{k^{100}}$
15. $2\sqrt{4x^2y^7z^9}$
16. $r^2\sqrt{40r^4s^{12}}$

17. $\sqrt{3} \bullet \sqrt{6}$

18. $2\sqrt{12} \bullet \sqrt{2}$

19. $100\sqrt{5} \bullet -2\sqrt{6}$

20. $8 \bullet 3\sqrt{2}$

21. $(3\sqrt{6})(9\sqrt{2})(4\sqrt{10})$

22. $-\sqrt{11} \bullet -4\sqrt{121}$

23. $x\sqrt{3x} \bullet x\sqrt{5x}$

24. $\sqrt{x} \bullet \sqrt{y} \bullet \sqrt{z^2}$

25. $2\sqrt{3} \bullet 4\sqrt{5} \bullet 6\sqrt{7}$