Probability Review Topics

Permutation: Order matters	Combination: Order doesn't matter

<div align="center">

Permutation: Order matters   Combination: Order doesn't matter

nPr   nCr

Distinguishable Permutations   Fundamental Counting Principle

$\dfrac{(total\ items)!}{(repeat)!(repeat)!...}$

*using all items, fill in blanks, multiply

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Probability: Likelihood of an event occurring to total number of outcomes

Independent/Dependent (AND) vs. Mutually Inclusive/Exclusive (OR)

AND…MULTIPLY	OR…ADD
Independent	**Mutually Exclusive**
One event does not affect the outcome of the second event Ex: Flipping a coin and rolling a die P(A) x P(B)	The events cannot happen at the same time Ex: Being a boy vs being a girl P(A)+P(B)
Dependent	**Mutually Inclusive**
One event affects the outcome of the second event Ex> picking a card and picking a second card without replacing the first card P(A) x P(B) (after A happens)	The events can happen at the same time Ex: Being a boy and having blue eyes P(A)+ P(B) – P(A and B)

Binomial Probability
(exactly, at least, at most)

The probability of an event, p, occurring exactly r times
$$_nC_r \cdot p^r \cdot q^{n-r}$$ n = number of trials r = number of specific events you wish to obtain p = probability that the event will occur q = probability that the event will **not** occur ($q = 1 - p$, the *complement* of the event)

Expected Value
Make a table like this…

Probability			
Outcome			

E=(outcome)(probability)+(outcome)(probability)+…

*Don't forget the amount you pay to play is a negative *All probability adds up to 1

Geometric Probability

$$Probability = \frac{Area\ of\ Desired\ Region}{Total\ Area}$$

Circle:	Square/Rectangle:	Triangle:
$A = \pi r^2$	$A = l \cdot w$	$A = \frac{1}{2}bh$ or $A = \frac{1}{2}absinC$

Probability Review

1. Three fair coins are tossed. What is the probability that there is only one head?

   Sample Space: HHH, HHT, HTH, (HTT,) THH, (THT) (TTH) TTT $\frac{3}{8}$

2. A poker hand consisting of five cards is dealt from a deck of 52 cards. Find the probability of all five cards being spades.

   $$\frac{_{13}C_5}{_{52}C_5} = \frac{1287}{2598960} = .000495 \quad OR \quad \frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} \cdot \frac{10}{49} \cdot \frac{9}{48}$$

3. A jar contains 5 red, 4 green balls and 2 yellow balls. Find the probability of the given event.

   a. A green ball is drawn and replaced and then a yellow ball is drawn.

   $$\frac{4}{11} \cdot \frac{2}{11} = \frac{8}{121}$$

   b. A red ball is drawn and then a white ball is drawn without replacing the first ball.

   $$\frac{5}{11} \cdot \frac{0}{11} = 0$$

   c. A red ball is drawn and then another red ball is drawn without replacing the first ball.

   $$\frac{5}{11} \cdot \frac{4}{10} = \frac{20}{110} = \frac{2}{11}$$

4. A researcher claims that she has taught a monkey to spell the word LEOPARD using six **7** wooden letters D, A, R, L, O, P, and E. If the monkey has not actually learned anything and is merely arranging the blocks randomly, then

   a. What is the probability that he will spell the word correctly

   $$\frac{1}{7!} = \frac{1}{5040}$$

   b. What is the probability that he will spell the word correctly three consecutive times?

   $$\frac{1}{5040} \cdot \frac{1}{5040} \cdot \frac{1}{5040} = \left(\frac{1}{5040}\right)^3 = .00000000000078$$

5. A card is drawn, a die is rolled and a coin is tossed. Find the probability of each outcome.

   a. The queen of hearts, a two and a tails,

   $$\frac{1}{52} \cdot \frac{1}{6} \cdot \frac{1}{2} = \frac{1}{624}$$

   b. A face card, a number more than three, and a heads.

 $$\frac{12}{52} \cdot \frac{3}{6} \cdot \frac{1}{2} = \frac{36}{624} = \frac{3}{52}$$

6. In our class, what is the probability of picking a student that is male or picking a student that has blonde hair?

   $$P(male) + P(blonde) - P(male \cap blonde)$$

7. You are taking a true/false quiz with 9 questions. What is the probability of getting all 9 questions correct?

   $$\left(\frac{1}{2}\right)^9 = \frac{1}{512}$$

8. Point P in square ABCD is chosen at random. Find the probability that point P is in squatter AXYZ.
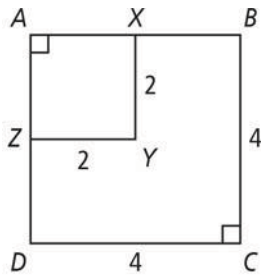
$\frac{1}{4}$

Geo Prob

9. The probability of a student passing my class is 0.8. If 5 students are selected at random, what is the probability that at least 4 students will pass the class.

Binomial Prob: $_5C_4 \cdot (.8)^4 \cdot (.2)^1$ or $(.8)^4 = .4096$

10. At the State Fair there is a booth where people can throw dimes onto a table that has dishes on it. Suppose that the chance that a dime lands in a dish is 0.3. Suppose you play the game 10 times.

    a. What is the probability that you will throw a dime in a dish at least 8 times?

Binom Prob

    b. What is the probability that you will throw a dime in the dish exactly 7 times?

11. You won a trip to Vegas and are feeling pretty lucky! You decide to play a game where you roll a die 8 times. What is the probability that

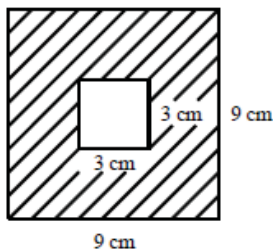   a. You roll a number greater than 4 at most 2 times?

Binom Prob

    b. You roll an even number exactly 3 times?

    c. You roll a 6 at least 7 times?

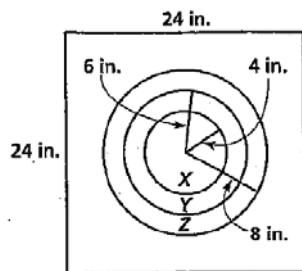**12.** Find the probability of landing in the shaded region.

Geo Prob

13. Use the following dart board to answer the questions below:

Geo Prob

24 in.

6 in.   4 in.

24 in.

X
Y
Z

8 in.

a. If a dart hits the board, find the probability that it will land in region X

b. If a dart hits the board, find the probability that it will land in region Z

c. If a dart hits the board, find the probability that it will NOT hit any of the circles.

14. Three digit numbers are formed using the digits 2, 4, 5 and 8, with repetition of digits allowed. How many such numbers are there that are:
less than 800? *first number ≠ 8*   even? *last digit ≠ 5*

$\underline{3} \cdot \underline{4} \cdot \underline{4} = 48$ $\underline{4} \cdot \underline{4} \cdot \underline{3} = 48$

15. A fancy restaurant offers 6 appetizers, 4 types of salads, 12 main courses and 3 desserts. In how many different ways can a customer order a meal from this restaurant?

$$6 \cdot 4 \cdot 12 \cdot 3 = 864$$

16. Mr. Davis has 24 people, 16 female and 8 male, trying out for the school play. He wants to choose a leading male, a leading female, a supporting male, supporting female and 6 extras - 2 males and 4 females. In how many ways can the cast be chosen?

$$8 \cdot 16 \cdot 7 \cdot 15 \cdot {}_6C_2 \cdot {}_{14}C_4 = 201,801,600$$

Leading male   Lead female   Support male   Support female   Male Extra   Female Extra

17. Art, Becky, Carl, Denise, and Ed all want to go see Carrie Underwood in concert. However, they only have 3 tickets. How many ways can they choose the three who get to go to the concert?

$${}_5C_3 = 10$$

18. A volleyball team has nine players. In how many ways can a starting line-up be chosen consisting of two forward players and three defense players?

* Not a good question

$${}_9C_2 \cdot {}_7C_3 = 1260$$

19. Part I of an exam has 5 multiple choice questions with 4 choices for each question. In how many different ways can this part of the exam be completed?

$$\underline{4} \cdot \underline{4} \cdot \underline{4} \cdot \underline{4} \cdot \underline{4} = 1024$$

~~20.~~ *should be question 19*

20. Refer to question 6, assume you randomly guess the answers at the answers. What is the probability that you do not get all answers correct?

$$\frac{1023}{1024} = .999$$

21. A child's game has a spinner which has spaces labeled 1 to 9 and all of the spaces are of equal size. What is the probability that the spinner stops on an odd number or a number greater than 6?

$$\frac{5}{9} + \frac{3}{9} - \frac{2}{9} = \frac{6}{9} = \frac{2}{3}$$

22. A bowl contains 5 oranges and 4 tangerines. Noelle randomly selects one, puts it back, and then selects another. What is the probability that both selections are tangerines?

$$\frac{4}{9} \cdot \frac{4}{9} = \frac{16}{81}$$

23. A ball is randomly selected from an urn that contains five red balls, three white balls, and one yellow ball. Find the probability that the ball is red or yellow.

$$\frac{5}{9} + \frac{1}{9} = \frac{6}{9} = \frac{2}{3}$$

24. Two balls are randomly selected from an urn that contains five red balls, three white balls, and one yellow ball. If the ball is not replaced after the first is selected, find the probability that both are red.

$$\frac{5}{9} \cdot \frac{4}{8} = \frac{20}{72} = \frac{5}{18}$$

25. You pay $0.50 to draw one card from a deck of cards. If it is an ace, you win $10; if it is a face card, you win $1; otherwise, you lose. What is the expected value of this game? Is this game fair?

	ACE	FACE	None
Prob	4/52	12/52	36/52
Outcome	+9.50	+.50	-.50

$$E = 9.5\left(\frac{4}{52}\right) + .5\left(\frac{12}{52}\right) + (-.5)\left(\frac{36}{52}\right)$$
$$= \boxed{\$.5} \quad \text{Not Fair} \ (E \neq 0)$$

26. A $1 bet is made to draw three cards from a standard deck of 52 cards. If all three cards are face cards (12 face cards in a deck), then you win $4.00. Find the expectation of this game and ~~explain~~ if you should play or not.

$$P(3 \text{ face}) = \frac{12}{52} \cdot \frac{11}{51} \cdot \frac{10}{50} = \frac{11}{1105}$$

$$E = 3\left(\frac{11}{1105}\right) + (-1)\left(\frac{1094}{1105}\right) = \boxed{-\$0.96}$$

27. Suppose you surveyed the students in your class on their favorite juice flavors. Their choices were 6 apples, 10 orange, 1 grapefruit, and 3 mangos. Record the *experimental* probability for each flavor.

Apples: $\frac{6}{20}$

Orange: $\frac{10}{20}$

GF: $\frac{1}{20}$

Mango: $\frac{3}{20}$

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