Factoring Polynomials: Notes

Greatest Common Factor

**** First step for all factoring methods

1. Find GCF of all terms
2. Factor (divide) the GCF from each term

$24a^3b^2 - 4a^2b^2 - 16a^2b^4$

$4a^2b^2(6a - 1 - 4b^2)$

Factor by Grouping

Type 1: 1. All terms share a common factor 2. Factor the common factor from each term	$\mathbf{3(x+y) + a(x+y)}$ $(x+y)(3+a)$
Type 2: (4 terms) 1. Group the 1st two terms and the last two terms. 2. Factor out the GCF from each grouping Notice: Both have the same factor now 3. Factor out the common term	$\mathbf{(a^2b + 3a^2) + (2b + 6)}$ $a^2(b+3) + 2(b+3)$ $(b+3)(a^2+2)$

Factoring a Trinomial: Leading Coefficient is 1: $x^2 + bx + c$

1. Find 2 numbers ($n_1 and\ n_2$) that MULTIPLY to c and ADD to b 2. List the factors if needed 3. Factor into two binomials $(x + n_1)(x + n_2)$	$\mathbf{x^2 + 12x + 20}$ $(x+10)(x+2)$

$x^2 + 2x - 35$ $(x+7)(x-5)$	$\boldsymbol{x^2 - 15x + 56}$ $(x-8)(x-7)$

Factoring a Trinomial: Leading Coefficient not 1: $\boldsymbol{ax^2 + bx + c}$

Trial and Error 1. Factors of a need to be the coefficients of the binomials$(a_1x + n_1)(a_2x + n_2)$ 2. Factors of c will be the constants n_1, n_2 3. Guess and check	$\boldsymbol{2x^2 + 7x + 5}$ $(2x+5)(x+1)$
Factor by Grouping 1. Multiply $a \cdot c$ 2. Find two numbers (n_1 *and* n_2) that MULTIPLY to $a \cdot c$ and ADD to b 3. Split up the bx term into $n_1x + n_2x$ creating four total terms 4. Use Factor By Grouping: Group first two terms and last two terms 5. Factor out GCF of each group. 6. Factor out the common factor	$\boldsymbol{4x^2 - 21x - 18}$ $4x^2 - 24x + 3x - 18$ $4x(x-6) + 3(x-6)$ $(x-6)(4x+3)$

Swing Method
(aka Bottoms Up aka Slide and Divide aka VA Tech Method aka the Illegal Way)

1. Multiply a and c (Swing)
2. Rewrite as $x^2 + bx + ac$
3. Factor the trinomial
4. Divide factors by a
5. Simplify the fractions
6. Swing back the denominators to the front.

$15x^2 + 7x - 4$

$x^2 + 7x - 60$

$(x + \frac{12}{15})(x - \frac{5}{15})$

$(x + \frac{4}{5})(x - \frac{1}{3})$

$(5x + 4)(3x - 1)$

Use any method: $10x^2 - 11x - 6$

$x^2 - 11x - 60$

$(x - \frac{15}{10})(x + \frac{4}{10})$

$(2x - 3)(5x + 2)$

Use any method: $6x^2 - 21x - 45$

$3(2x^2 - 7x - 15)$

$3(x^2 - 7x - 30)$

$3(x - \frac{10}{2})(x + \frac{3}{2})$

$3(x - 5)(2x + 3)$

Special Factoring

Difference of Squares: $a^2 - b^2$

1. Square root $a^2 = a$
2. Square root $b^2 = b$
3. $(a + b)(a - b)$

$\boldsymbol{x^2 - 49}$

$(x-7)(x+7)$

$\boldsymbol{81y^2 - 100x^2}$

$(9y - 10x)(9y + 10x)$

Perfect Square Trinomials

1. First term is a perfect square
2. Last term is a perfect square
3. Middle term is twice the first times last

$$\boldsymbol{A^2 + 2AB + B^2} = (\boldsymbol{A + B})(\boldsymbol{A + B}) = (\boldsymbol{A + B})^2$$

$$\boldsymbol{A^2 - 2AB + B^2} = (\boldsymbol{A - B})(\boldsymbol{A - B}) = (\boldsymbol{A - B})^2$$

$\boldsymbol{x^2 - 10x + 25}$

$(x-5)^2$

$\boldsymbol{9x^2 + 72x + 144}$

$(3x + 12)^2$

$= 9(x + 4)^2$

Multiple Techniques

1. Make to check for other factors after using each technique. 2. Often times using one method will turn unveil another factorable situation. 3. The problem isn't complete unless it is fully factored.	$y^4 - 16$ $(y^2+4)(y^2-4)$ $(y^2+4)(y-2)(y+2)$
$2x^3 - 8x$ $2x(x^2-4)$ $2x(x-2)(x+2)$	$x^4 - 1$ $(x^2+1)(x^2-1)$ $(x^2+1)(x+1)(x-1)$

Practice:

$15x^3 + 9x^2$ $3x^2(5x+3)$	$-6x^4y^2 + 18x^2y - 24xy$ $-6xy(x^3y-3x+6)$

$3ax + 6ay + 4x + 8y$ $3a(x+2y) + 4(x+2y)$ $(x+2y)(3x+4)$	$2ac + 4ax - 5c - 10x$ $2a(c+2x) - 5(c+2x)$ $(c+2x)(2a-5)$
$9x^2 - 16y^4$ $(3x+4y^2)(3x-4y^2)$	$2xy^2z^3 - 18x^3z$ $2xz(y^2z^2 - 9x^2)$ DoS $2xz(yz-3x)(yz+3x)$
$3x^6 - 27y^6$ $3(x^6 - 9y^6)$ DoS $3(x^3-3y^3)(x^3+3y^3)$	$x^2 - 7x + 12$ $(x-3)(x-4)$

$x^2 - 4x - 12$ $(x-6)(x+2)$	$12x^3 - 5x^2 - 3x$ $x(12x^2 - 5x - 3)$ $x(x^2 - 5x - 36)$ $x(x - \frac{9}{12})(x + \frac{4}{12})$ $x(x - \frac{3}{4})(x + \frac{1}{3})$ $x(4x-3)(3x+1)$
$-4x^2 + 23x + 6$ $-(4x^2 - 23x - 6)$ $-(x^2 - 23x - 24)$ $-(x - \frac{24}{4})(x + \frac{1}{4})$ $-(x-6)(4x+1)$	$4x^2 - 18x + 14$ $2(2x^2 - 9x + 7)$ Both Prime $2(2x-7)(x-1)$

Practice for Factoring All Types Name _____ pd. ___ Date _____

1. Common factors:

a. $4x^2y^3 + 6x^5y^2 – 10x^4y^7$ b. $12x^5y^2 – 18x^2y + 21x^4y^3$ c. $2ax + 6xc + ba + 3bc$

d. $5a^3 – 3a^2b + 6b^3 – 10ab^2$ e. $5xy + 6m + 10ym + 3x$ f. $7mp^2 + 2np^2 – 7mq^2 -2nq^2$

2. Trinomials:

a. $x^2 – 5x – 24$ b. $x^2 + 6x – 16$ c. $x^2 + 10x + 9$ d. $x^2 – 17x + 30$

e. $3x^2 + 7x + 2$ f. $4x^2 – 11x + 7$ g. $6x^2 – 7x – 10$ h. $3x^2 – 2x – 8$

3. Difference of 2 squares:

a. $x^2 – 49$ b. $x^2 – 81$ c. $4x^2 – 25y^2$ d. $9x^2 – 16$

4. Mixture with common factors to pull out first:

a. $2x^2 + 10x – 28$ b. $3x^2 – 12x – 63$ c. $4x^2 – 10x - 24$

d. $15x^2 + 6x - 9$ e. $4x^2 - 100$ f. $3x^2 + 15$

a. $x^2 + 12x - 45$ b. $6x^2 + 5x - 6$ c. $2x^2 - 98$

d. $6mx - 4m + 3rx - 2r$ e. $2a^3b^2 - 16a^2b^3 + 8ab$ f. $5ax^2 - 20ay^2$

g. $9x^2 - 25y^2$ h. $x^2 - x - 56$ i. $8x^2 + 10x - 12$

j. $ay + 4p + 4a + yp$ k. $x^2 - 9x - 36$ l. $3x^2 + 8x + 4$