Algebra 1 Name: _____

Geometric Sequences

Warm Up:

Recall the formula for an Arithmetic Sequence:

$a_n = a_1 + (n-1)d$

a_n = nth term
a_1 = 1st term
d = com dif

1. Find the next terms and the equation for the sequence:

-37, -23, -9, 5, 19, 33
+14 +14 +14

$a_1 = -37$
$d = 14$

$a_n = -37 + (n-1)(14)$
$= -37 + 14n - 14$
$a_n = 14n - 51$

a) Find the 150th term:

$a_{150} = 14(150) - 51 = 2049$

2. Find the arithmetic means between

937, 840, 743, 646, 549, 452

(1, 937) (6, 452)

$\frac{452 - 937}{6 - 1} = \frac{-485}{5} = -97$

Geometric Sequences

Definition: In a **geometric sequence,** the ratio of any term to the previous term is constant. This constant is called the **common ratio** and is denoted by r.

EXAMPLE: Decide whether the sequence is geometric check: $r = \frac{a_n}{a_{n-1}}$

A) 1, 2, 6, 24, 120

+1 +4 +18 +96

$\frac{2}{1} = 2$ $\frac{6}{2} = 3$ $\frac{24}{6} = 4$ $\frac{120}{24} = 5$ No!

B) 81, 27, 9, 3, 1

÷3 ÷3 ÷3 ÷3

$r = \frac{27}{81} = \frac{9}{27} = \frac{3}{9} = \boxed{\frac{1}{3}}$

C) -4, 8, -16, 32, -64

×-2 ×-2 ×-2 ×-2

$r = -2$

Arithmetic $(a_n = a_1 + (n-1)d)$

Rule for a Geometric Sequence:

The nth term of a geometric sequence with first term a_1 and common ratio r is given by:

nth term → $a_n = a_1 r^{n-1}$ ← 1st Term, common rat.

EXAMPLE: Find the formula for the nth term of the geometric sequence $6, -2, \frac{2}{3}, \ldots$. What is the tenth term?

$a_1 = 6$ $r = \frac{a_2}{a_1} = \frac{-2}{6} = \frac{-1}{3}$ $\boxed{a_n = 6\left(-\frac{1}{3}\right)^{n-1}}$

$r = -\frac{1}{3}$

$a_{10} = 6\left(-\frac{1}{3}\right)^{10-1}$

$a_{10} = -\frac{2}{6561}$

EXAMPLE: The 3rd term of a geometric sequence is $\frac{16}{3}$ and the 7th term of the sequence is $\frac{256}{243}$. Find the 9th term in the sequence

$a_n = a_1 r^{n-1}$

EXAMPLE: Find the geometric mean of 10 and 40.

$40 = 10 r^{3-1}$

10 20 40

a_1 a_3

$r = 2$

EXAMPLE: Insert three geometric means between 7 and 567.

7 21 63 189 567

a_1 a_5

$\frac{567}{7} = \frac{7}{7} r^{5-1}$

$(81)^{\frac{1}{4}} = (r^4)^{\frac{1}{4}}$

$r = 3$

There are two formulas for finding any term in a geometric sequence. Either formula will work…the choice of which one to use depends on what you're given to work with.

Recursive Formula

$$a_n = a_{n-1} \cdot r$$

Explicit Formula

$$a_n = a_1 \cdot r^{n-1}$$

Example 2

Using the Formulas

Find the 19th term given a_{18} = 1,420,541,793, r = 3

$a_{19} = a_{18} \cdot r$

$= (1.4 \times 10^9) 3 = 4.2 \times 10^9$

Find the 19th term given a_1 = 11, r = 3

$a_{19} = 11(3)^{19-1}$

$a_{19} = 4.2 \times 10^9$

Example 3

Writing Formulas

Write an explicit and recursive formula for each sequence.

a_1 =10, r = 2

$a_n = a_{n-1} \cdot 2$ - recursive

$a_n = 10(2)^{n-1}$ - explicit

3, -3, 3, -3, …

$a_1 = 3 \quad r = -1$

$a_n = a_{n-1}(-1)$

$a_n = 3(-1)^{n-1}$