

Sequences 3.5

Quiz Tomorrow:

- Determine whether an equation is linear
- Write linear equations in standard form
- Find x and y intercepts
- Find rate of change from table (linear or not)
- Find slope of a line given two points
- Direct Variation

Warm Up:

1. Find Slope of the line that passes through $(-2, 2)$ and $(-6, 4)$

$$M = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 2}{-6 - (-2)} = \frac{2}{-4} = -\frac{1}{2}$$

2. Suppose y varies directly with x. Write a direct variation equation that relates If $y = 6$ when $x = 10$, find x when $y = 18$.

$$y = kx$$

$$\frac{6}{10} = k \left(\frac{10}{10} \right)$$

$$k = \frac{6}{10} = \frac{3}{5}$$

$$y = \frac{3}{5}x$$

$$18 = \frac{3}{5}x$$

$$x = 30$$

3. Find the rate of change represented by the table:

	+3	+6	+3
X	1	4	10
Y	2	6	14
	+4	+8	+4

$$R.o.C: \frac{4}{3}, \frac{8}{6} = \frac{4}{3}, \frac{4}{3} \rightarrow \text{Linear}$$

4. Is the equation linear? If so write in standard form. $\left(-\frac{3}{4}y = -\frac{1}{4}x - 1 \right) \times 4$

$$-3y = -x - 4$$

$$+x \quad +x$$

$$x - 3y = -4$$

Air-ith-met-ic

Arithmetic Sequences Notes

Definition: A sequence is an: *ordered set of numbers*

Definition: An arithmetic sequence is a sequence in which each term after the first is found by adding a constant, called the common difference (d) to the previous term.

Example 1: Fill in the next 3 terms. What is the common difference?

$$2, 5, 8, 11, 14, 17, \underline{20}, \underline{23}, \underline{26} \quad d = 3$$

$+3 \quad +3 \quad +3 \quad +3 \quad +3$

Example 2: Fill in the next 3 terms. What is the common difference?

$$55, 49, 43, \underline{37}, \underline{31}, \underline{25} \quad d = -6$$

$-6 \quad -6 \quad -6$

Term Notation: 2, 5, 8, 11, ... $d = 3$

<u>Terms</u>	<u>symbol</u>	<u>Symbolic Not.</u>	<u>Number</u>
First Term	a_1	a_1	2
Second Term	a_2	$a_1 + d$	$2 + 3 = 5$
Third Term	a_3	$a_2 + d$ $(a_1 + d) + d$ $a_1 + 2d$	$2 + 2(3) = 8$
Fourth Term	a_4	$a_1 + 3d$	$2 + 3(3) = 11$

ellipses $\{ \}$: $\rightarrow$ pattern continues

n^{th} term $a_n = a_1 + (n-1)d$ $2 + (n-1)(3)$

Formula for writing an equation of an arithmetic sequence:

The n th term a_n of an arithmetic sequence with first term a_1 and common difference d is given by:

$$a_n = a_1 + (n - 1)d$$

Where n is any positive integer. (* n stands for the number of terms in the sequence)

Example: Write an equation for the n th term of the arithmetic sequence

8, 17, 26, 35, ...

METHOD 1 - need a_1 and d

$$a_1 = 8$$

$$d = 9$$

$$n = n$$

$$a_n = a_n$$

$$a_n = 8 + (n - 1)(9)$$

$$a_n = 8 + 9n - 9$$

$$a_n = 9n - 1$$

METHOD 2

$$\begin{array}{cccc} 8, & 17, & 26, & 35, \\ a_1 & a_2 & a_3 & a_4 \end{array}$$

$$\begin{array}{ccc} (1, 8) & (2, 17) & (3, 26) \\ x_1 y_1 & x_2 y_2 & x_3 y_3 \end{array}$$

$$m = \frac{17 - 8}{2 - 1} = 9$$

Example: Use either method to find the equation for the n th term of $-8, -6, -4, \dots$

$$a_1 = -8$$

$$d = 2$$

$$a_n = a_1 + (n-1)d$$

$$a_n = -8 + (n-1)(2)$$

$$= -8 + 2n - 2$$

B) Find $a_{13} \rightarrow 13^{\text{th}}$ Term

$$n = 13$$

$$a_n = 2n - 10$$

$$a_{13} = 2(13) - 10$$

c) Find the 120^{th} term.

$$n = 120$$

$$a_{13} = 16$$

$$a_{120} = 230$$

Example: Find the 4 arithmetic means between 16 and 91. (This means the 4 missing terms between 16 and 91). *Need to find d .

$$16, \underline{31}, \underline{46}, \underline{61}, \underline{76}, 91$$

$+15 \quad +15 \quad +15 \quad +15 \quad +15$

$(1, 16) \qquad (6, 91)$

$$d = \frac{91 - 16}{6 - 1} = \frac{75}{5} = 15$$

$$a_1 = 16$$

$$a_6 = 91$$

$$n = 6$$

$$a_n = a_1 + (n-1)d$$

$$91 = 16 + (6-1)d$$

$$75 = 5d$$

$$d = 15$$

Example: Given $a_1 = 21$ and $a_5 = 45$, find the equation of the arithmetic sequence and find the 3 arithmetic means between them.

$$\begin{array}{ccccccc} 21, & 27, & 33, & 39, & 45 \\ (1, 21) & \underbrace{\hspace{10em}} & (5, 45) \end{array}$$

$$\frac{45 - 21}{5 - 1} = 6$$

$$a_n = 21 + (n - 1)(6)$$

$$a_n = 21 + 6n - 6$$

$$a_n = 6n + 15$$

Example: Given 52, 46, 40, 34... Write a formula and find the 97th term.

$$a_n = 52 + (n - 1)(-6)$$

$$a_n = -6n + 58$$

$$a_{97} = -524$$

Given $a_6 = 74$ and $a_{13} = 151$. Find the first term

$$\begin{array}{ccc} & 74 & 151 \\ \hline (1, a_1) & (6, 74) & (13, 151) \end{array}$$

$$d = \frac{151 - 74}{13 - 6} = 11$$

$$11 = \frac{74 - a_1}{6 - 1}$$

$$11 = \frac{74 - a_1}{5}$$

$$\begin{array}{r} 55 = 74 - a_1 \\ - 74 \quad 74 \\ \hline \end{array}$$

$$-19 = -a_1$$

$$\boxed{a_1 = 19}$$

$$a_n = a_1 + (n-1)d$$

$$151 = a_1 + (13-1)(11)$$

$$151 = a_1 + 12(11)$$

$$\begin{array}{r} 151 = a_1 + 132 \\ - 132 \quad 132 \\ \hline \end{array}$$

$$\boxed{a_1 = 19}$$

52, 39, 26, ..., -260, -273, -286

① Write formula $d = -13$ $a_1 = 52$

② Find 12th Term

③ Which term is -182

④ How many terms are in the sequence

$$① a_n = 52 + (n-1)(-13)$$

$$\boxed{a_n = -13n + 65}$$

$$② a_{12} = -13(12) + 65 = -91$$

$$③ a_n = -182$$

$$-182 = -13n + 65$$

$$\textcircled{4} \quad n = ?$$

$$-286 = -13n + 65$$