$$y = 2|x|$$
$$y = |2x|$$

Ha1   Name: _____

Transformations Review

RULES FOR TRANSFORMATIONS OF FUNCTIONS
If $f(x)$ is the original function, $a > 0$ and $c > 0$:

Function	Transformation of the graph of $f(x)$
$f(x) + c$	Shift $f(x)$ upward c units
$f(x) - c$	Shift $f(x)$ downward c units
$f(x + c)$	Shift $f(x)$ to the left c units
$f(x - c)$	Shift $f(x)$ to the right c units
$-f(x)$	Reflect $f(x)$ in the x-axis
$f(-x)$	Reflect $f(x)$ in the y-axis
$a \cdot f(x),\ a > 1$	Stretch $f(x)$ vertically by a factor of a.
$a \cdot f(x),\ 0 < a < 1$	Shrink $f(x)$ vertically by a factor of a.
$f(ax),\ a > 1$	Shrink $f(x)$ horizontally by a factor of $\frac{1}{a}$.
$f(ax),\ 0 < a < 1$	Stretch $f(x)$ horizontally by a factor of $\frac{1}{a}$.

Ex: List the transformations of $f(x)$ for each of the following functions.

$g(x) = 2f(x)$	$h(x) = f(x - 3)$	$j(x) = f\left(\frac{1}{3}x\right)$
Vertical Stretch by factor of 2	Right 3	Horizontal Stretch by a factor 3
$k(x) = f(-x)$	$m(x) = f(x) - 3$	$n(x) = f(x + 3) + 3$
Reflect over y	Down 3	Left 3 up 3

Order of Transformations:

Does the order of transformations matter?

1. Start with the function $f(x) = x^2$
    a. Shift up 7 units, then right 2 units.

$$f(x) = x^2 \longrightarrow \underset{up\ 7}{x^2 + 7} \longrightarrow \underset{right\ 2}{(x-2)^2 + 7}$$

    b. Shift right 2 units, then up 7.

$$x^2 \longrightarrow \underset{right\ 2}{(x-2)^2} \longrightarrow (x-2)^2 + 7$$

    c. Do parts (a) and (b) give the same equation?

    yes

2. Start with the function $f(x) = \sqrt{x}$
    a. Stretch vertically by a factor of 2, then shift downward 5 units.

$$f(x) = \sqrt{x} \longrightarrow \underset{stretch\ by\ 2}{g(x) = 2\sqrt{x}} \longrightarrow \boxed{\underset{Down\ 5}{h(x) = 2\sqrt{x} - 5}}$$

    b. Shift downward 5 units, then stretch vertically by a factor of 2.

$$\sqrt{x} \longrightarrow \underset{Down\ 5}{\sqrt{x} - 5} \longrightarrow \underset{\substack{Stretch \\ by\ 2}}{2(\sqrt{x} - 5)} = 2\sqrt{x} - 10$$

    c. Do parts (a) and (b) give the same equation

    No

When does order matter?

- when there are multiple Vertical/Horizontal movements.

What order should be use?

Order of Operations
① Horizontal Shifts
② Stretch or Compress
③ Reflections
④ Vertical Shift

Horizon S/C

$$a \cdot f\left(b\left(x+c\right)\right)+d$$

vertical Stretch or Compress

H Shift

V shift

For each of the following, list the parent function, the transformations, and the "new origin". Then graph the equation. (HINT: It might help to graph the Parent Function.) Then determine the domain & range.

7) $y = 3(x-2)^2 + 3$

PF: x^2

Transformations:

Right 2

Vert Stretch 3

Up 3

New Origin: $(2, 3)$

8) $y = -4\sqrt{x+2} - 1$

PF: $\sqrt{x}$

Transformations:

Left 2

V. Stretch by 4

Reflect x

Down 1

New Origin: _____

$(-2, -1)$

9) $y = -|x+2| - 5$

PF: $|x|$

Transformations:

Left 2

Reflect over x

Down 5

New Origin: $(-2, -5)$

10) $y = -\frac{1}{2}x^3 + 1$

PF: x^3

Transformations:

Vertical Compress by $\frac{1}{2}$

Reflect over x

Up 1

New Origin: $(0, 1)$

Write the equation of each parent function given the certain conditions. (

11) Absolute Value: Compression of $\dfrac{3}{4}$, shifted up 4 units, shifted left 3 units.

$$y = \frac{3}{4}|x + 3| + 4$$

12) Cubic: Shifted 3 down, shifted 1 left, and reflected across the x-axis.

$$y = -(x + 1)^3 - 3$$

13) Quadratic: Reflected across x-axis, stretch of 3, shifted down 2.

$$y = -3x^2 - 2$$
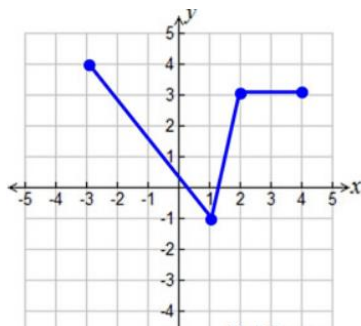
14) Exponential: Shifted 6 right, 7 up and stretch of 5.

$$y = 5 \cdot 2^{x-6} + 7$$

15) Square root: Compression of $\dfrac{5}{7}$, shifted right 4, shifted up 1
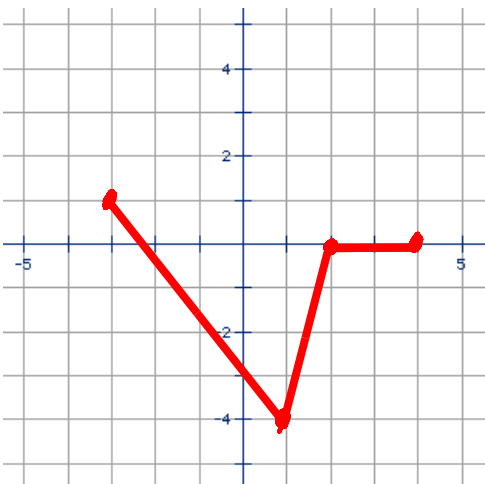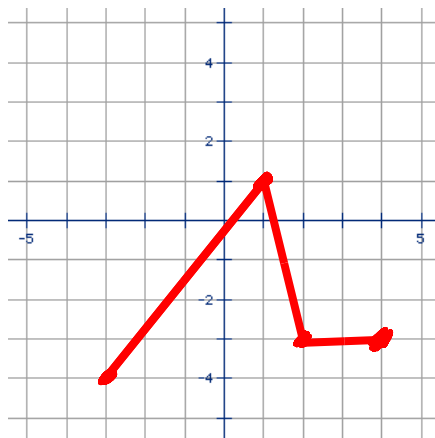
$$y = \frac{5}{7}\sqrt{x-4} + 1$$
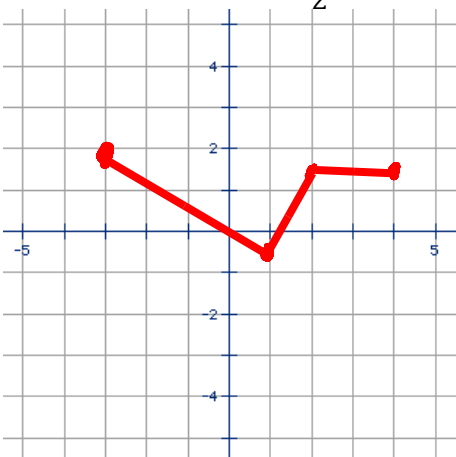
Practice:
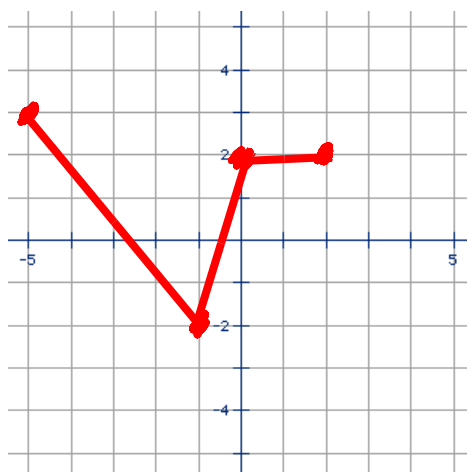
Given: f(x)

Graph the following:

$g(x) = f(x) - 3$	$g(x) = -f(x)$

$g(x) = \dfrac{1}{2}f(x)$	$g(x) = f(x+2) - 1$

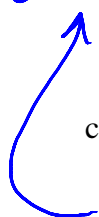Consider the graph of $y = (x + 5)^2 + 1$ Make one change in the original equation of $f(x)$ so that the graph:

    a. Opens downward.

              b. Is fatter (compressed).

$$y = -(x+5)^2 + 1$$

$$y = \frac{1}{2}(x+5)^2 + 1$$

    c. Has x-intercepts.

              d. Is moved to the lett.

$$y = \frac{1}{2}(x+6)^2 + 1$$

Given the function: $f(x) = (x + 3)^2 + 5$

Write the function that:

    a) Shifts f(x) 10 units right

$$g(x) = (x - 7)^2 + 5$$

    b) Shifts f(x) 5 units down

$$g(x) = (x + 3)^2$$

    c) Reflects over y axis

$$g(x) = (-x + 3)^2 + 5$$

    d) Stretches by a factor of 2

$$g(x) = 2(x + 3)^2 + 5$$